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Little supersymmetry and the supersymmetric little hierarchy problem

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Abstract: The current experimental lower bound on the Higgs mass significantly restricts the allowed parameter space in most realistic supersymmetric models, with the consequence that these models exhibit significant fine-tuning. We propose a solution to this 'supersymmetric little hierarchy problem'. We consider scenarios where the stop masses are relatively heavy - in the 500 GeV to a TeV range. Radiative stability of the Higgs soft mass against quantum corrections from the top quark Yukawa coupling is achieved by imposing a global SU(3) symmetry on this interaction. This global symmetry is only approximate - it is not respected by the gauge interactions. A subgroup of the global symmetry is gauged by the familiar SU(2) of the standard model. The physical Higgs is significantly lighter than the other scalars because it is the pseudo-Goldstone boson associated with the breaking of this symmetry. Radiative corrections to the Higgs potential naturally lead to the right pattern of gauge and global symmetry breaking. We show that both the gauge and global symmetries can be embedded into a single SU(6) grand unifying group, thereby maintaining the prediction of gauge coupling unification. Among the firm predictions of this class of models are new states with the quantum numbers of 10 and $\overline{10}$ under SU(5) close to the TeV scale. The Higgs mass is expected to be below 130 GeV, just as in the MSSM.

Keywords: Global Symmetries, Supersymmetric Standard Model, GUT, Higgs Physics.

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JHEP10(2004)036

Contents

1. Introduction

Supersymmetry is perhaps the most attractive solution of the hierarchy problem. Among its many appealing features is the observed unification of the standard model gauge couplings [1] normalized as in SU(5)[2] in the Minimal Supersymmetric standard model or 'MSSM'. However, most realistic supersymmetric models today suffer from a naturalness problem, called the 'supersymmetric little hierarchy problem'. This problem arises because in most of the parameter space of these theories the Higgs mass lies below the current experimental lower bound. As a consequence of this these theories are typically fine-tuned at the level of a few percent, rendering them highly unnatural.

Let us attempt to understand in more detail the origin of the supersymmetric little hierarchy problem. At tree level in the MSSM, the mass of the lightest neutral Higgs is bounded by M_Z. Therefore if the lightest neutral Higgs is to be heavy enough to avoid the current experimental bound, there must be significant quantum corrections to the Higgs potential, and in particular to the Higgs quartic coupling. Loops involving the scalar superpartners of the top quark, the stops, do indeed generate such a correction but only if the stops are relatively heavy. However, loops involving the stops also give quantum corrections to the Higgs soft mass parameter, and if the stops are heavy these corrections are too large and must be cancelled to within a few percent against the μ term to obtain correct electroweak symmetry breaking. This leads to significant fine-tuning. Therefore we see that the source of the problem is that the stop masses are being required to meet two contradictory criteria

- they must be large to generate a sizable quartic correction to the Higgs potential
- they must be small to avoid generating sizable logarithmic corrections to the Higgs soft mass parameter.

The fine-tuning is the result of an attempt to balance these two effects. Although both contributions arise from loops involving the top Yukawa couplings, the correction to the quartic arises mainly from loop momenta below the stop mass, while the unwanted correction to the soft mass parameter arises mainly from loop momenta much larger than the scale of the soft masses. Therefore if the Higgs mass can be protected against radiative corrections from scales significantly higher than the superpartner masses, the supersymmetric little hierarchy problem can be solved.

More insight into the problem may be gained by considering the manner in which supersymmetry addresses the hierarchy problem. We start with the observation that at the one loop level there are in fact three separate hierarchy problems in the standard model -

- quadratic corrections to the Higgs mass squared associated with the standard model gauge couplings
- quadratic corrections to the Higgs mass squared associated with the Higgs self coupling
- quadratic corrections to the Higgs mass squared associated with its Yukawa couplings to the standard model fermions.

In supersymmetric models the first two of these are closely related because the Higgs quartic term arises from the D-terms of the gauge multiplets. However they are indeed distinct from the third. This can be seen from the fact that quadratic divergences arising from the standard model Yukawa couplings are cancelled by the scalar superpartners of the fermions while divergences arising from the Higgs gauge and self interactions are cancelled by the gauginos and Higgsinos. The source of the supersymmetric little hierarchy problem is that the bound on the Higgs mass implies that the stop masses must be heavy. This then creates a fourth, much milder hierarchy problem.

- logarithmic corrections to the Higgs mass squared associated with the Yukawa couplings to the top quark, and proportional to the stop mass squared

This fourth hierarchy problem is clearly more closely related to the last in the list of three above than to the first two. For example, by keeping the gauginos light it is possible to address the first pair of hierarchy problems without addressing the second pair.

JHEP10(2004)036

In this paper we propose a new solution to the supersymmetric little hierarchy problem. We consider a scenario where the stop masses are relatively heavy, and lie in the 500 GeV to 1 TeV range. In order to achieve radiative stability against the quantum corrections from the top Yukawa coupling from scales above the superpartner masses a global symmetry, which for concreteness we will take to be $\mathrm{SU}(3)_{\mathrm{L}}$, is imposed on this interaction. The global symmetry is only approximate - it is not respected by the gauge interactions. A subgroup of the global symmetry is gauged by the $\mathrm{SU}(2)_{\mathrm{L}}$ in the standard model. The Higgs fields of the MSSM emerge from multiplets which transform as a 3 and $\bar{3}$ under the global symmetry. One linear combination of the two $\mathrm{SU}(2)_{\mathrm{L}}$ doublets in this 3, $\bar{3}$ pair, which is the physical Higgs, is protected against quantum corrections from the Yukawa couplings because it is the pseudo-Goldstone boson associated with the breaking of this global symmetry. This model is in the spirit of 'little Higgs' theories ([3], [4], see also [5]), which solve the little hierarchy problem by generating a light standard model Higgs as the pseudo-Goldstone boson of an approximate global symmetry.[1]. A clear and concise introduction to these theories may be found in [7].

If at some higher scale where the supersymmetry breaking masses are generated the global symmetry is exact, the scalar masses at this scale will respect SU(3). When renormalization group evolved down to the scale of the scalar masses they will still respect the symmetry to a very good approximation because at one loop nearly all logarithmically enhanced corrections to the SU(3) invariant form are proportional to the square of the electroweak gaugino masses, which are typically small. At one loop the only SU(3) violating correction that does not fall into this category is proportional to the sum of all the scalar masses squared weighted by hypercharge. However, this quantity vanishes or is small in a large class of models, either as a consequence of the pattern of supersymmetry breaking, or as a consequence of the way in which the model is embedded in a unified theory. Since the form of the Higgs potential at the scale of the scalar masses is approximately SU(3) symmetric, the standard model Higgs, which is the pseudo-Goldstone associated with the breaking of this global symmetry, will be significantly lighter than the other scalars and can be made to naturally acquire a weak scale VEV.

How is the SU(3) global symmetry broken? The simplest possibilty is that it is broken radiatively in exactly the same manner as conventional electroweak symmetry breaking in the MSSM driven by the top Yukawa coupling. A quartic restoring potential for the Higgs of the right form may be obtained by incorporating into the model a U(1) gauge symmetry that commutes with the global SU(3). The right alignment pattern is favored because of the D terms associated with the standard model gauge interactions that violate the global symmetry. The subsequent breaking of the electroweak symmetry group down to U(1) electromagnetism also occurs naturally. Quantum corrections from stop loops to the Higgs quartic potential now allow the mass of the lightest neutral Higgs to be above the experimental bound without significant fine tuning.

[1]Earlier work on using global symmetries to protect against radiatively induced scalar masses in a locally supersymmetric context can be found in [6].

JHEP10(2004)036

Is the prediction of gauge coupling unification maintained in this class of models? We show that it is straightforward to embed the global SU(3) in an SU(6) grand unifying group. The Standard Model quarks, leptons and gauge fields then have their conventional embedding in the SU(5) subgroup of SU(6). In order to realize the SU(3) symmetry at low scales while preserving the prediction of gauge coupling unification, a pair of fields with the quantum numbers of a 10 and $\overline{10}$ under SU(5) must be present at or close to the weak scale. This is a firm prediction of this model.

What are the characteristic features of such a 'little supersymmetric' theory ? We expect the stops to be relatively heavy, above about 500 GeV. The constraints on the rest of the superpartner spectrum are fewer, but natural electroweak symmetry breaking requires that the electroweak gauginos be lighter than the stops. There is an additional Z' gauge boson associated with the new U(1) gauge symmetry that couples to the standard model fields, and which typically has a mass in the 300 GeV to a TeV range. There are also new states with masses of order a TeV associated with the global SU(3) symmetry of the top Yukawa. These have vector-like charges under the standard model gauge symmetry, and come in complete SU(5) multiplets in order to preserve unification. In contrast to other proposed solutions of the supersymmetric little hierarchy problem, the mass of the lightest neutral Higgs is expected to be below 130 GeV, just as in the MSSM.

While recently there has been considerable attention focused on potential solutions of the supersymmetric little hierarchy problem [8], [9], the approach we follow here differs significantly from that of other authors. Most recent work relies on altering the form of the Higgs potential close to the weak scale with a view to invalidating the MSSM bound on the Higgs mass. For example, by generating additional quartic terms beyond those in the MSSM the Higgs mass can be raised above 130 GeV ameliorating the fine tuning problem. For early work in this direction see, for example [10]. In contrast, we attempt to make the mass of the physical Higgs insensitive to loop corrections from higher scales. For an alternative recent approach where superconformal symmetry is used to suppress the Higgs mass parameters relative to the other soft parameters, see [9].

2. A minimal model

2.1 The global symmetry

The Yukawa coupling of the top quark to the up-type Higgs in the MSSM has the familiar form

$$(3,2)_{Q_3}(1,2)_{H_u}(\bar{3},1)_{t^c}\,, \tag{2.1}$$

where the numbers in brackets indicate the quantum numbers of the various fields under $\mathrm{SU}(3)\times\mathrm{SU}(2)_\mathrm{L}$. For simplicity, we have suppressed the hypercharge quantum numbers. Here, following the usual convention, Q_3 represents the third generation left handed quarks, t^c represents the third generation left handed antiquark and H_u is the up-type Higgs. We wish to suitably extend this interaction to make it invariant under a global SU(3) symmetry, which we denote by G, of which the familiar $\mathrm{SU}(2)_\mathrm{L}$ is a gauged subgroup. This can be

JHEP10(2004)036

done by extending

$$(3,2)_{Q_3}(1,2)_{H_u}(\bar{3},1)_{t^c} \rightarrow (3,\bar{3})_{\hat{Q}_3}(1,3)_{\hat{H}_u}(\bar{3},1)_{t^c}\,, \tag{2.2}$$

where the second number in each bracket on the right hand side of the arrow now indicates the transformation properties of each field under the new global SU(3) symmetry G. Here

$$\hat{Q}_3 = \left(Q_3, \overline{T^c}\right) \tag{2.3}$$

$$\hat{H}_u = (H_u, S_u)\,, \tag{2.4}$$

where S_u and $\overline{T^c}$ are the new states we have added to the theory to make it invariant under G. S_u is a singlet under the standard model gauge group while the quantum numbers of $\overline{T^c}$ are such that it is vector-like with respect to t^c. We also embed the down-type Higgs H_d into a $\bar{3}$ representation of the SU(3) global symmetry

$$\hat{H}_d = (H_d, S_d)\,, \tag{2.5}$$

where S_d is a singlet under the standard model gauge group. We further demand that the soft masses for $\hat{H}_u$ and $\hat{H}_d$ as well as the μ and $B\mu$ terms have a form which respects the global symmetry G at some high scale where they are generated. However the $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ gauge interactions do not respect the global symmetry, and further we do not require that the Yukawa couplings of the bottom quark or the lighter fermions respect the global symmetry either, since they are relatively small. Then, since the SU(3) violating loop corrections to the Higgs potential from the gauge and Yukawa couplings are relatively small, if S_u and S_d acquire VEVs that break the approximate global symmetry one linear combination of H_u and $H_d{}^*$ will remain light as the pseudo-Goldstone boson associated with the breaking of the approximate symmetry. This is the physical Higgs field. We define f as the scale at which the global symmetry is broken.

$$f = \sqrt{\langle S_u \rangle^2 + \langle S_d \rangle^2}\,. \tag{2.6}$$

If the global symmetry G is broken radiatively as in the MSSM we expect that the scale f is not far from the scale of the stop masses, which we denote by $\hat{m}$. In particular, this will be the case if the coefficient of the quartic restoring term in the Higgs potential for S_u and S_d is of order one. In our models this restoring term arises from the D-term of a new U(1) gauge symmetry, which we denote by $\mathrm{U(1)_E}$. The charges under $\mathrm{U(1)_E}$ are such that it commutes with the global symmetry G, which then implies that the standard model fields are charged under it. We can estimate f more precisely as being of order $\hat{m}/\hat{g}$, where $\hat{g}$ is gauge coupling strength of $\mathrm{U(1)_E}$. Then radiative corrections to the Higgs soft masses from scales below f that violate the symmetry G are not large enough to significantly affect the pseudo-Goldstone nature of the light Higgs.

There is a natural hierarchy between the scale f at which the global symmetry is broken and the scale at which electroweak symmetry is broken due to the difference in the sizes of the terms in the Higgs potential which respect the global SU(3) and those terms which violate the global SU(3). All the terms in the effective theory for the pseudo-Goldstone field below the scale $\hat{m}$ arise from relatively small loop effects that violate the global symmetry. It is therefore these small effects which determine the manner in which electroweak symmetry is broken. In the following subsections we explain in more detail how this happens.

Anomaly cancellation requires that there be an additional field T^c with the same gauge quantum numbers as t^c. In order to ensure that there are no unwanted light states with masses below the weak scale we add to the theory a mass term of the form $\int d^2\theta M T^c \overline{T^c}$. Although this term violates the global symmetry G, provided that the mass M is less than or of order f, the global symmetry breaking scale, the SU(3) violating corrections to the Higgs potential from scales below f will not be large. The physical left-handed third generation antiquark of the standard model is a linear combination of t^c and T^c, with the exact ratio determined by the relative sizes of f and M.

2.2 The pattern of symmetry breaking

Under what circumstances does this model give the pattern of symmetry breaking we seek, with $\langle S_u \rangle, \langle S_d \rangle \neq 0$, $\langle H_u \rangle, \langle H_d \rangle \neq 0$ and $\langle S_u \rangle > \langle H_u \rangle$, $\langle S_d \rangle > \langle H_d \rangle$? In order to understand this we consider the potential for the Higgs sector, $V_{\rm tot}$. Now $V_{\rm tot}$ can be broken up into two parts, one part which respects the global SU(3) and which we denote by $V_{\rm G}$, and another which breaks the global SU(3) and which we denote by $V_{\rm B}$.

$$V_{\rm tot} = V_{\rm G} + V_{\rm B}\,. \tag{2.7}$$

The forms of $V_{\rm G}$ and $V_{\rm B}$ are:

$$\begin{aligned} V_{\rm G} &= \left(\hat\mu^2 + \hat m_u^2\right)|\hat H_u|^2 + \left(\hat\mu^2 + \hat m_d^2\right)|\hat H_d|^2 + \left(\hat B\, \hat H_u\, \hat H_d + \text{h.c.}\right) + \\ &\quad + \frac{\hat g^2}{8}\left(|\hat H_u|^2 - |\hat H_d|^2\right)^2 \qquad (2.8) \\ V_{\rm B} &= \left(2\delta\mu\, \hat\mu + \delta\mu^2 + \delta {m_u}^2\right)|H_u|^2 + \left(2\delta\mu\, \hat\mu + \delta\mu^2 + \delta {m_d}^2\right)|H_d|^2 + \\ &\quad + \left(\delta B H_u H_d + \text{h.c.}\right) + \frac{{g_2}^2 + {g_1}^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{{g_2}^2}{2}|H_u^* H_d|^2\,, \qquad (2.9) \end{aligned}$$

where we have assumed for simplicity that all parameters in $V_{\rm tot}$ are real. The terms involving the supersymmetric parameters μ and $\delta\mu$ above emerge from the superpotential terms below which are generalizations of the μ term of the MSSM.

$$W_{\rm tot} = \hat\mu\, \hat H_u \hat H_d + \delta\mu\, H_u H_d\,. \tag{2.10}$$

By assumption the dimensionful parameters in $V_{\rm G}$, which are expected to be of order $\hat m$, are significantly larger than those in $V_{\rm B}$. In the next sub-section it will be seen that this assumption is consistent with the sizes of the radiative corrections that violate the global SU(3), so that in most of parameter space $|\delta m^2| \ll |\hat m^2|$.

Stability of the Higgs potential $V_{\rm tot}$ for large field values requires

$$2\,\hat\mu^2 + \hat{m_u}^2 + \hat{m_d}^2 > 2\,|B| \tag{2.11}$$

$$2\,\left(\hat\mu + \delta\mu\right)^2 + \hat{m_u}^2 + \delta {m_u}^2 + \hat{m_d}^2 + \delta {m_d}^2 > 2\,|B + \delta B|\,. \tag{2.12}$$

We now argue that in regions of parameter space where both $\hat{m}_u^2 < 0$ and $\delta {m_u}^2 < 0$, but with $|\delta {m_u}^2| \ll |\hat{m}_u^2|$ we get the pattern of symmetry breaking we require. We first consider minima of $V_{\rm G}$, since the dimensionful parameters in $V_{\rm B}$ are significantly smaller. Now the global SU(3) symmetry of $V_{\rm G}$ implies that it has both minima with $[\langle S_u\rangle, \langle S_d\rangle \neq 0, \langle H_u\rangle, \langle H_d\rangle = 0]$ as well as minima with $[\langle S_u\rangle, \langle S_d\rangle = 0, \langle H_u\rangle, \langle H_d\rangle \neq 0]$. However the latter vacua are strongly disfavored by the D terms of SU(2) $\times$ U(1) in $V_{\rm B}$. Therefore $[\langle S_u\rangle, \langle S_d\rangle \neq 0, \langle H_u\rangle, \langle H_d\rangle = 0]$ is an approximate solution to the potential $V_{\rm tot}$, if the dimensionful terms in $V_{\rm B}$ are ignored. In order to determine the pattern of electroweak symmetry breaking, we expand about this vacuum and determine the effective theory for the light fields below the scale $\hat{m}$. This is an expansion in the small parameter $(\delta m/\hat{m})^2$. The only field with mass significantly smaller than $\hat{m}$ is one linear combination of H_u and H_d^*, which is the pseudo-Goldstone associated with the breaking of the SU(3) global symmetry. We denote this light field by $H_{\rm L}$ and the orthogonal linear combination which is heavy by $H_{\rm H}$.

$$H_{\rm L} = \sin\beta \, H_u \; + \; \cos\beta \, H_d^* \tag{2.13}$$

$$H_{\rm H} = -\cos\beta \, H_u \; + \; \sin\beta \, H_d^* \,. \tag{2.14}$$

The potential for $H_{\rm L}$ in the effective theory below the scale $\hat{m}$ has the simple form:

$$V_{\rm L} = m_{\rm L}^2 |H_{\rm L}|^2 + \frac{{g_2}^2 + {g_1}^2}{8} \cos^2 2\beta \, |H_{\rm L}|^4 \,. \tag{2.15}$$

Here the parameter $m_{\rm L}^2$ is given by:

$$m_{\rm L}^2 = \left(2\delta\mu \, \hat{\mu} + \delta\mu^2 + \delta {m_u}^2 \sin^2\beta + \delta {m_d}^2 \cos^2\beta\right) + \; \delta B \sin 2\beta \,. \tag{2.16}$$

The condition for electroweak symmetry breaking is that $m_{\rm L}^2 < 0$. Now if $\hat{\mu}$ is not very large and if $\tan^2\beta \gg 1$ this is equivalent to requiring $\delta m_u^2 < 0$. Therefore the criterion for getting the right pattern of symmetry breaking is that $\hat{m}_u^2 < 0$, $\delta m_u^2 < 0$ with $|\hat{m}_u^2| \gg |\delta m_u^2|$.

Since the quartic terms in the potential for $H_{\rm L}$ depend on the gauge coupling strengths of $\mathrm{SU}(2)_{\rm L}$ and $\mathrm{U}(1)_{\rm Y}$ we can determine the mass of the lightest neutral Higgs at tree level to be $m_{\rm Z} \cos 2\beta$. However just as in the MSSM the quartic term in the Higgs potential receives significant corrections from loops involving the top Yukawa coupling. The relevant part of the superpotential is the SU(3) invariant top Yukawa coupling, which we denote by λ and the mass term that decouples the extra colored states.

$$\int d^2\theta \, \lambda \left(\, Q_3 H_u t^c + \overline{T^c} S_u t^c \, \right) + \; M \, T^c \, \overline{T^c} \,. \tag{2.17}$$

We are particularly interested in the limit $M > \lambda \langle S_u\rangle$ because the physical top Yukawa is given by

$$\lambda_t = \lambda \, \frac{M}{\sqrt{M^2 \; + \; \lambda^2 \langle S_u\rangle^2}} \,. \tag{2.18}$$

This implies that in order to get a physical top Yukawa of order one we typically require $M > \lambda \langle S_u\rangle$. The bound on the mass of the extra U(1) gauge boson typically requires that $\langle S_u\rangle \geq 2.5\,\mathrm{TeV}$ [11], which then implies that in the region of interest $\langle S_u\rangle > \hat{m}$

as well. Therefore we have the hierarchy $M > \lambda \langle S_u \rangle > \hat{m}$. For the effective theory below the scale M, the relevant part of the superpotential is then exactly as in the MSSM.

$$\int d^2\theta \, \lambda_t \, Q_3 H_u t^c \,. \tag{2.19}$$

This gives the familiar logarithmically enhanced one-loop contribution to the Higgs quartic coupling

$$\Delta \, V_{\rm B} = \frac{3\lambda_t^4}{16\pi^2} |H_u|^4 \ln \left(\frac{m_{\tilde{t}}^2}{m_t^2} \right) . \tag{2.20}$$

The effect of this, as in the MSSM, is to raise the mass of the lightest neutral Higgs above $\mathrm{M_Z}$.

2.3 Radiative stability of the global symmetry

We now investigate the logarithmically enhanced radiative corrections to the Higgs potential using the renormalization group equations with a view to understanding whether they give rise to the pattern of symmetry breaking discussed in the previous section, and whether the global symmetry is stable under quantum corrections. In particular, since the forms of all logarithmically enhanced contributions to the Higgs potential, both $\mathrm{SU(3)_G}$ preserving and $\mathrm{SU(3)_G}$ violating, can be determined from the renormalization group equations, this is a test of the validity of treating the physical Higgs as a pseudo-Goldstone boson. For simplicity we ignore all superpotential couplings in the analysis apart from the top Yukawa and and the μ terms. We also ignore some small SU(3) invariant effects, such as radiative corrections from the $\mathrm{U(1)_E}$ gauge coupling, since these do not affect the parameters in the pseudo-Goldstone potential at one loop and are otherwise not large enough to affect our conclusions.

We parametrize the superpotential as

$$\int d^2\theta \, \left(\lambda_u Q_3 H_u t^c + \lambda_s \overline{T^c} S_u t^c + \hat{\mu} \; S_u S_d \; + \mu H_u H_d \right) . \tag{2.21}$$

At some ultraviolet scale Λ the SU(3) global symmetry is exact, which implies $\lambda_u = \lambda_s$ and $\hat{\mu} = \mu$. However radiative corrections from scales below Λ will in general alter these relations. The renormalization group equations for the Yukawa couplings are

$$\frac{d\,\lambda_u}{d\,t} = \frac{1}{16\pi^2} \lambda_u \left(6{\lambda_u}^2 + {\lambda_s}^2 - \frac{16}{3}{g_3}^2 - 3{g_2}^2 - \frac{13}{9}{g_1}^2 \right) \tag{2.22}$$

$$\frac{d\,\lambda_s}{d\,t} = \frac{1}{16\pi^2} \lambda_s \left(2{\lambda_u}^2 + 5{\lambda_s}^2 - \frac{16}{3}{g_3}^2 - \frac{16}{9}{g_1}^2 \right) . \tag{2.23}$$

We see that these couplings receive corrections from $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ gauge loops that violate the SU(3) global symmetry, so that at low scales we do not expect them to have the same value. Close to the weak scale we expect that each coupling will be driven to a fixed point value just as in the MSSM; however the two fixed point values will now no longer be the same. The difference between the fixed point values is given by

$$\lambda_u^2 - \lambda_s^2 = \frac{1}{4} \left(3g_2^2 - \frac{1}{3} g_1^2 \right) . \tag{2.24}$$

JHEP10(2004)036

Since this difference is small compared to the individual couplings themselves global SU(3) is still a good approximate symmetry of the Yukawa couplings at low energies.

What about the renormalization group equations for $\hat{\mu}$ and μ? These have the form:

$$\frac{d\,\hat{\mu}}{d\,t} = \frac{1}{16\pi^2}\hat{\mu}\left(3{\lambda_s}^2\right) \tag{2.25}$$

$$\frac{d\,\mu}{d\,t} = \frac{1}{16\pi^2}\mu\left(3{\lambda_u}^2 - 3{g_2}^2 - {g_1}^2\right). \tag{2.26}$$

In contrast to the Yukawa couplings we see that the difference $\delta\mu$ between $\hat{\mu}$ and μ at low energies, although loop suppressed is logarithmically enhanced, and can be comparable to the absolute values of $\hat{\mu}$ and μ themselves. We conclude that global SU(3) is typically not a good approximate symmetry of the dimensionful mass terms for the Higgs fields in the superpotential. We therefore require that these terms be small compared to $\hat{m}$ in order that the approximate global symmetry of the Higgs potential $V_{\rm tot}$ is preserved.

We now consider the dominant terms in the renormalization group equations for the soft masses of H_u and S_u:

$$\frac{d\,m_s^2}{d\,t} = \frac{1}{16\pi^2}\left[3\lambda_s^2\left(2m_{\tilde{t}}^2 + 2m_s^2 + 2m_{\tilde{T}}^2\right)\right] \tag{2.27}$$

$$\frac{d\,m_u^2}{d\,t} = \frac{1}{16\pi^2}\left[3\lambda_u^2\left(2m_{\tilde{t}}^2 + 2m_u^2 + 2m_{\tilde{Q}}^2\right) - 6g_2^2\,M_2^2\, - 2g_1^2\,M_1^2\right]. \tag{2.28}$$

In these equations M_2 and M_1 are the masses of the SU(2) and U(1) gauginos. We see from this that m_s^2 and m_u^2 are driven negative so that radiative breaking of the SU(3) global symmetry occurs exactly as in the MSSM through loops involving the top Yukawa coupling. As shown in the previous subsection the VEV f naturally lies along S_u and S_d, the electroweak singlet directions, because of the D terms of the $\mathrm{SU(2)_L \times U(1)_Y}$ gauge group.

What about electroweak symmetry breaking? Does it also naturally occur? As explained in the previous subsection the criterion for this is that $\delta m_u^2 = m_u^2 - m_s^2 < 0$. The one loop renormalization group equations for the soft masses above would seem to indicate that $\delta {m_u}^2$ is in fact greater than zero, due to the net positive contribution from gauge and gaugino loops to the mass of H_u. However, a closer inspection reveals that this is not the case. Since at low scales λ_u and λ_s differ by the amount shown in (2.24), and in particular $\lambda_u > \lambda_s$ we see that there is a net two loop contribution to the mass splitting that drives $\delta {m_u}^2 < 0$. If the stop masses are larger than the $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$ gaugino masses the two loop contribution is dominant so that electroweak symmetry breaking can take place. This is indeed the case for most realistic supersymmetric spectra.

In these models there are restrictions from naturalness on the scale at which the soft superpartner masses are generated. We denote this scale by $\Lambda_{\rm soft}$. Then the renormalization group equations imply that at low scales

$$|\delta m_u^2| \approx \frac{3}{4\pi^2}\left(\lambda_u^2 - \lambda_s^2\right)\,\hat{m}^2 \log\left(\frac{\Lambda_{\rm soft}}{\hat{m}}\right) \approx \frac{\hat{m}^2}{50}\log\left(\frac{\Lambda_{\rm soft}}{\hat{m}}\right). \tag{2.29}$$

If the logarithm above becomes of order 10 the hierarchy between δm and $\hat{m}$ is affected. This shows that in this scenario low values of the scale $\Lambda_{\rm soft}$ are preferred by naturalness. This class of theories includes models of low energy supersymmetry breaking such as gauge mediation [12], where $\Lambda_{\rm soft}$ is the scale of the messenger masses.

The fact that the standard model matter fields are charged under $\mathrm{U(1)_E}$ also puts important restrictions on the soft scalar masses. The reason is that the non-zero D-term of $\mathrm{U(1)_E}$ will in general give a correction of order $\hat{m}^2$ to the soft mass squared of each scalar superpartner, the sign of which may be positive or negative, depending on the charge of the field under $\mathrm{U(1)_E}$. We therefore require that those fields which get a negative correction from the D-term of $\mathrm{U(1)_E}$ receive a larger positive contribution to their soft masses from supersymmetry breaking. This places restrictions on the manner in which supersymmetry is mediated to the visible sector fields. In the absence of fine-tuning this generally implies that all the soft scalar masses are at least of order $\hat{m}$, except the physical Higgs.

The constraint that the μ terms be small, in conjunction with the constraints (2.12) on the parameters of the Higgs potential also places restrictions on the manner in which supersymmetry breaking is communicated to the visible sector. In particular there should not be too much of a hierarchy between the stop masses and the up and down Higgs masses at $\Lambda_{\rm soft}$, so that even when electroweak symmetry is broken the sum of the up-type and down-type soft Higgs mass squared remains positive.

3. Grand unification

3.1 Embedding of the gauge and global symmetries

We now explain how the framework we have outlined in the previous section can be successfully incorporated into a supersymmetric grand unified theory, thereby maintaining the prediction of coupling constant unification. The approach we will take is to identify the global symmetry with a subgroup of a larger grand unifying group, which we take to be SU(6). The reason for this choice is that since SU(6) contains $\mathrm{SU(3) \times SU(3) \times U(1)}$ as maximal subgroups it is natural to identify one of these two SU(3) groups with color and the other with the global symmetry we require the theory to possess. But why then is one of these two SU(3) groups completely gauged while only an $\mathrm{SU(2)_L \times U(1)_Y}$ subgroup of the remaining $\mathrm{SU(3) \times U(1)}$ is gauged? This can be understood as a consequence of the pattern of symmetry breaking. We consider a scenario where there are two separate effects which each break SU(6), but in two completely different breaking patterns.

- The first breaks $\mathrm{SU(6) \to SU(5)}$
- The other breaks $\mathrm{SU(6) \to SU(3) \times SU(3) \times U(1)}$.
- Their net result is $\mathrm{SU(6) \to SU(3) \times SU(2)_L \times U(1)_Y}$.

For example the first pattern can be obtained if the SU(5) singlet components of a 6, $\bar{6}$ pair acquire VEVs, while the second can be obtained if an adjoint of SU(6), a 35, acquires a VEV along $\mathrm{Diag}(1,1,1,-1,-1,-1)$. We denote these VEVs by $\langle 6\rangle, \langle\bar{6}\rangle$ and

$\langle 35 \rangle$ respectively. For now we will assume that this pattern of VEVs is responsible for the breaking of SU(6), although our results are in fact more general. If the Yukawa coupling of the top quark is screened from the first of the two effects above that break SU(6) it will be invariant under the SU(3) global symmetry up to loop corrections. This is what we require for our scenario to be viable.

How can the standard model quarks, leptons and Higgs fields be embedded in the SU(6) group? The SU(6) representations we require are the 6 and $\bar{6}$, the 15, which is the two index antisymmetric tensor, and the 20, which is the three index antisymmetric tensor. Under the SU(5) subgroup of SU(6) these representations decompose in the following way:

$$6 \to 5 + 1 \tag{3.1}$$

$$\bar{6} \to \bar{5} + 1 \tag{3.2}$$

$$15 \to 5 + 10 \tag{3.3}$$

$$20 \to 10 + \overline{10} \tag{3.4}$$

$$35 \to 24 + 5 + \overline{5} + 1\,. \tag{3.5}$$

In an SU(5) grand unified theory one standard model family of quarks and leptons emerges from the $\bar{5}$ and 10 representations. The simplest generalization of this to SU(6) involves two $\bar{6}$ representations and a 15; this particular combination is anomaly free. Once SU(6) is broken down to SU(5) the 5 from the 15 and one of two $\bar{5}$'s from the $\bar{6}$'s are now vector-like under the unbroken group and can be given a mass and decoupled, along with the singlets, leaving the familiar chiral $\bar{5}$ and 10 at low energies. The interactions that do this are of the form

$$\langle\bar{6}\rangle\, 15\, \bar{6}\,, \qquad \frac{\langle 6\rangle \,\langle 6\rangle}{\Lambda}\, \bar{6}\, \bar{6}\,. \tag{3.6}$$

Here Λ is an ultraviolet scale, which may either be the Planck scale or a scale associated with the mass of a singlet that is integrated out to obtain this operator.

What about the Higgs fields? In an SU(5) grand unified theory the up and down Higgs fields emerge from the doublets in the 5 and $\bar{5}$ representations respectively. In the most straightforward generalization of this to SU(6) the Higgs fields emerge from the SU(2) doublets in a 6, $\bar{6}$ pair. This however leads to a difficulty. While the down-type Yukawa couplings can be easily generalized from SU(5) to SU(6)

$$\bar{5}_{\bar{\mathrm{H}}}\; 10\; \bar{5} \to \bar{6}_{\bar{\mathrm{H}}}\; 15\; \bar{6} \tag{3.7}$$

there is no way to write an SU(6) invariant up-type Yukawa coupling with this minimal choice of matter and Higgs representations. However once SU(6) is broken to SU(5) it is possible to write such couplings. For example, the familiar up-type Yukawa coupling of SU(5) can be generalized to SU(6)

$$5_{\mathrm{H}}\; 10\; 10 \to \frac{\langle 6\rangle}{\Lambda}\, 6_{\mathrm{H}}\; 15\; 15\,. \tag{3.8}$$

Here Λ is either the Planck scale or a scale associated with the mass of a vector-like $15, \overline{15}$ pair that is integrated out to obtain this operator [13]. While this is satisfactory for the first two generations it does not serve our purposes as regards the third generation. In particular, since this operator breaks SU(6), an up-type mass term of this form does not preserve the SU(3) global symmetry that we wish the top Yukawa to respect. While there may be more than one possible solution to this problem the approach we will follow here is to introduce into the theory a 20 dimensional representation of SU(6), which is a three-index antisymmetric tensor. Then the up-type Yukawa coupling of SU(5) can be generalized to SU(6) as [14]

$$5_{\rm H}\; 10\; 10 \to 6_{\rm H}\; 15\; 20\,. \tag{3.9}$$

Since this interaction is SU(6) invariant it naturally respects the SU(3) global symmetry. However, the model now contains an extra 10 and $\overline{10}$ of SU(5) beyond the fields in the MSSM.

In conventional grand unified models based on the SU(6) group once SU(6) is broken to SU(5) or its subgroups the unwanted $\overline{10}$ (of SU(5)) coming from the 20 pairs up with one linear combination of the 10's from the 15 and from the 20, acquires a mass and decouples. Therefore the physical third generation fields emerge from the orthogonal linear combination. The interactions that lead to this have the form

$$\int d^2\theta\; \langle 6\rangle\, 15\; 20 + 20\,\langle 35\rangle\, 20\,. \tag{3.10}$$

The net result is that at low energies we are left with precisely the familiar 3 generations which have the quantum numbers of $\bar{5}$ and 10 under SU(5) . This however does not serve our purpose, since decoupling the extra 10 and $\overline{10}$ destroys the global SU(3) symmetry of the top Yukawa coupling that we are trying to preserve. We therefore assume that such terms are forbidden until supersymmetry is broken, in analogy with the μ term of the MSSM, and that instead we have

$$\int d^2\theta\; \tilde{\mu}\, \frac{\langle 35\rangle}{\Lambda}\, 20\; 20\,, \qquad \int d^2\theta\; \tilde{\mu}\, \frac{\langle 6\rangle}{\Lambda}\, 15\; 20\,, \tag{3.11}$$

where $\tilde{\mu}$ is of order $\hat{m}$, and Λ is an ultraviolet scale which we assume is close to (but above) the unification scale. Then the additional 10 and $\overline{10}$ are present in the theory until the weak scale, and therefore the global SU(3) symmetry of the top Yukawa is maintained until then.

Therefore to summarize, the matter content of this class of unified models is given by 3 generations of $[\,\bar{6}, \bar{6}, 15\,]$ and a single 20, while the light Higgs fields emerge from a 6 and a $\bar{6}$. A subset of this matter content, constituting three generations of fields which transform as 5, $\bar{5}$ under SU(5) , along with six SU(5) singlets (two per generation), is vector-like under the standard model gauge group and decouples near the unification scale. However, another subset of this matter content, which has the quantum numbers of a 10 and $\overline{10}$ under SU(5) , is also vector-like under the standard model gauge group but remains light until the weak scale thereby preserving the global symmetry of the top Yukawa coupling.

In this scenario the pseudo-Goldstone nature of the physical Higgs can be understood as a consequence of the fact that the VEVs of $6_{\rm H}$ and $\bar{6}_{\bar{\rm H}}$ are aligned along the same direction as $\langle 6\rangle$ and $\langle\bar{6}\rangle$, while at the same time the potentials of $6_{\rm H}$ and $6_{\bar{\rm H}}$ are screened from $\langle 6\rangle$ and $\langle\bar{6}\rangle$. Therefore an SU(3) subgroup of the SU(6) gauge symmetry survives as an approximate global symmetry of the sector of the theory containing $\hat{H}_u$ and $\hat{H}_d$. This idea is similar in spirit to that of the little Higgs model of Kaplan and Schmaltz [4]. A major difference is the large hierarchy of scales between the two pairs of VEVs which makes logarithmically enhanced radiative corrections to the potential for the light Higgs fields important.

3.2 Realization of the global symmetry

We now study in more detail how the global symmetry is realized in this class of models, and in particular how the fermion masses emerge. For this purpose it is useful to understand how the various SU(6) representations decompose under the SU(3)×SU(3)×U(1) subgroup of SU(6) . For simplicity, in what follows we suppress the U(1) quantum numbers, except to distinguish between the two singlets of SU(3) × SU(3) in the 20. While one of these has the U(1) quantum numbers of τ^c the other has the quantum numbers of the complex conjugate of τ^c.

$$\begin{aligned} 6 &\to (3,1) + (1,3) && (3.12) \\ \bar{6} &\to (\bar{3},1) + (1,\bar{3}) && (3.13) \\ 15 &\to (3,3) + (\bar{3},1) + (1,\bar{3}) && (3.14) \\ 20 &\to (3,\bar{3}) + (\bar{3},3) + (1,1)_{\tau^c} + (1,1)_{\overline{\tau}} && (3.15) \\ 35 &\to (8,1) + (3,\bar{3}) + (\bar{3},3) + (1,8) + (1,1)\,. && (3.16) \end{aligned}$$

Consider first the top Yukawa coupling (3.9). Under SU(3) × SU(3) this decomposes as shown below

$$6_{\mathrm{H}}\ 15\ 20 \to \begin{cases} (1,3)_{\mathrm{H}}(\bar{3},1)(3,\bar{3}) \\ (1,3)_{\mathrm{H}}(3,3)(\bar{3},3) \\ (1,3)_{\mathrm{H}}(1,\bar{3})(1,1)_{\overline{\tau}}\,, \end{cases} \qquad (3.17)$$

where we have ignored the interactions of the color triplet Higgs, which are irrelevant to our discussion. The first of the three terms on the right hand side above is precisely the SU(3) invariant interaction (2.2) that we require. We must ensure that the form of this term is not disturbed by any SU(3) violating interaction except at loop level.

Consider now the interaction (3.6) which decouples the vector-like states in the $\bar{6}$'s and the 15 at the unification scale. Under SU(3) × SU(3) it decomposes as shown below.

$$\langle\bar{6}\rangle\ 15\ \bar{6} \to \begin{cases} \langle 1,\bar{3}\rangle\ (3,3)(\bar{3},1), \\ \langle 1,\bar{3}\rangle\ (1,\bar{3})(1,\bar{3})\,. \end{cases} \qquad (3.18)$$

Although this interaction removes states which are not in complete multiplets of the global SU(3), notice that it does not directly affect any of the states in the SU(3) invariant top Yukawa interaction, the first line on the right hand side of (3.17). However it does affect the interactions on the second and third lines on the right hand side of (3.17), which are no longer SU(3) × SU(3) invariant in the low energy effective theory, but instead transform as shown below under $SU(3) \times SU(2)_L$ of the standard model.

$$(1,3)_{\rm H}(3,3)(\bar{3},3) \rightarrow [1,2]_{\rm H}[3,2][\bar{3},1] + [1,1]_{\rm H}[3,2][\bar{3},2] \tag{3.19}$$

$$(1,3)_{\rm H}(1,\bar{3})[1,1]_{\bar{\tau}} \rightarrow [1,1]_{\rm H}[1,1][1,1]_{\bar{\tau}}\,. \tag{3.20}$$

Above and in what follows we use ordinary brackets to denote quantum numbers under $SU(3) \times SU(3)$ and square brackets to denote quantum numbers under $SU(3) \times SU(2)_L$. Since the interactions (3.19) and (3.20) violate the global SU(3) we require that they be small so that they do not significantly feed into the Higgs soft mass at loop level. However the SU(6) symmetry of the operator (3.9) that generates all these couplings would imply that all of the couplings in (3.17) have the same strength. We therefore require that there be significant contributions that violate SU(6) but respect $SU(3) \times SU(3)$ in the sector that generates the top Yukawa. These could arise from operators such as the one below.

$$\frac{\langle 35 \rangle}{\Lambda} 6\ 15\ 20 \rightarrow \begin{cases} (1,3)(\bar{3},1)(3,\bar{3}), \\ (1,3)(3,3)(\bar{3},3)\,, \\ (1,3)(1,\bar{3})(1,1)_{\bar{\tau}} \end{cases} \tag{3.21}$$

We see that this decomposes under $SU(3) \times SU(3)$ into the very same operators as in (3.17), but these now have different coefficients.

We now move on to consider the operators (3.11), which give masses of order $\hat{m}$ for the remaining light states which are vector-like under SU(5)

$$\int d^2\theta\, \tilde{\mu}\, \frac{\langle 35 \rangle}{\Lambda}\ 20\ 20 \rightarrow \begin{cases} \tilde{\mu}\ (3,\bar{3})(\bar{3},3), \\ \tilde{\mu}\ (1,1)_{\bar{\tau}}(1,1)_{\tau^c} \end{cases} \tag{3.22}$$

$$\int d^2\theta\, \tilde{\mu}\, \frac{\langle 6 \rangle}{\Lambda}\ 15\ 20 \rightarrow \begin{cases} \tilde{\mu}\ [\bar{3},1][3,1], \\ \tilde{\mu}\ [3,2][\bar{3},2], \\ \tilde{\mu}\ [1,1][1,1]_{\bar{\tau}}\,. \end{cases} \tag{3.23}$$

These masses are not invariant under the global SU(3). When S_u (which in our notation is $[1,1]_{\rm H}$) acquires a VEV we see from the form of the interactions (3.17), (3.22) and (3.23) that one linear combination of the $[\bar{3},1]$ from the 15 and the $[\bar{3},1]$ from the 20 gets a mass with the $[3,1]$ from the 20 leaving the orthogonal linear combination light. This light state is the physical t^c. It is straightforward to establish that from (3.19), (3.20), (3.22) and (3.23) that the physical Q_3 and τ^c are also linear combinations of states from the 15 and the 20.

We now come to the third generation Yukawa couplings in the down sector. These emerge from the operator (3.7), which decomposes under $SU(3) \times SU(3)$ as shown below

$$\bar{6}_{\bar{\rm H}}\ 15\ \bar{6} \rightarrow \begin{cases} (1,\bar{3})_{\bar{\rm H}}(3,3)(\bar{3},1)\,, \\ (1,\bar{3})_{\bar{\rm H}}(1,\bar{3})(1,\bar{3})\,. \end{cases} \tag{3.24}$$

As a consequence of the operator (3.6) which decouples states that are not in complete representations of the global SU(3), these interactions are not invariant under the global symmetry below the unification scale. At low energies they have their familiar standard model forms under $\mathrm{SU}(3) \times \mathrm{SU}(2)_{\mathrm{L}}$ as shown below.

$$(1, \bar{3})_{\bar{\mathrm{H}}}(3, 3)(\bar{3}, 1) \rightarrow [1, 2]_{\bar{\mathrm{H}}}[3, 2][\bar{3}, 1] \tag{3.25}$$

$$(1, \bar{3})_{\bar{\mathrm{H}}}(1, \bar{3})(1, \bar{3}) \rightarrow [1, 2]_{\bar{\mathrm{H}}}[1, 1]_{\tau^c}[1, 2]\,. \tag{3.26}$$

Although these couplings are not invariant under the global symmetry they are not large enough to significantly affect the SU(3) symmetric form of the top Yukawa.

What about the first two generations? How do their Yukawa couplings arise? The down-type masses arise from operators of the very same form (3.7) as for the third generation. At the unification scale all the states in these two generations which are vector-like or singlets under the standard model gauge group get a mass from the operator (3.6), so that at low energies only the standard model fields survive. The down-type interactions then take their familiar standard model form, exactly as in (3.25) and (3.26) above. The up-type Yukawa couplings however, in contrast to the top Yukawa, arise from the operator (3.8). This operator manifestly breaks the global SU(3) symmetry at the unification scale. However it is too small to significantly affect the SU(3) symmetric form of the top Yukawa coupling. It decomposes under $\mathrm{SU}(3) \times \mathrm{SU}(2)_{\mathrm{L}}$ precisely into the up-type Yukawa couplings of the standard model.

$$\frac{\langle 6 \rangle}{\Lambda}\, 6_{\mathrm{H}}\ 15\ 15 \rightarrow [1, 2]_{\mathrm{H}}[3, 2][\bar{3}, 1]. \tag{3.27}$$

3.3 A realistic unified model

We now construct a realistic unified model based on the ideas we have outlined. In order to solve the doublet-triplet splitting problem as in [15] we consider models with a compact fifth dimension where the breaking of $\mathrm{SU}(6) \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{U}(1)$ is realized by imposing appropriate boundary conditions on fields. The extra dimension is assumed to be extremely small, with size of order the unification scale, which is about $10^{15} - -10^{16}$ GeV. In contrast however, the breaking of SU(6) to SU(5) is realized by the expectation values $\langle 6 \rangle$ and $\langle \overline{6} \rangle$ of fields transforming in the fundamental and anti-fundamental representations of SU(6) . Provided that this theory is strongly coupled at the cutoff of the higher dimensional theory, we expect the coupling constants to unify at about the same level as in the MSSM [16].

We now consider the breaking of $\mathrm{SU}(6) \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{U}(1)$ in more detail. Explicitly, we start with a five dimensional SU(6) gauge theory compactified on an S^1/Z_2 orbifold. Compactification on S^1/Z_2 is obtained by identifying the fifth coordinate x_5 under the two operations $x_5 \rightarrow -x_5$ (reflection) and $x_5 \rightarrow x_5 + 2\pi R$ (translation). Then the physical space is the line interval between $x_5 = 0$ and $x_5 = \pi R$. We assume that there are branes at the orbifold fixed points $x_5 = 0$ and $x_5 = \pi R$ where fields can be localized. The boundary conditions are chosen so that the reflection symmetry breaks 5D N = 1 supersymmetry to 4D N = 1 supersymmetry while the translation symmetry breaks the gauge symmetry. Specifically we start with a five dimensional SU(6) gauge field $A_M \equiv A^a_M T^a, M = 1 \ldots 5, a = 1 \ldots 35$, and impose the following boundary conditions:

$$\begin{aligned} A_\mu(x^\mu, x_5) &= +A_\mu(x^\mu, -x_5) = Z\, A_\mu(x^\mu, x_5 + 2\pi R)\, Z^{-1}\,, \\ A_5(x^\mu, x_5) &= -A_5(x^\mu, -x_5) = Z\, A_5(x^\mu, x_5 + 2\pi R)\, Z^{-1}\,, \end{aligned} \tag{3.28}$$

where $\mu = 1 \ldots 4$ and $Z = \mathrm{diag}(+,+,+,-,-,-)$. The low-energy effective field theory in four dimensions contains the seventeen massless gauge bosons A_μ of the unbroken $\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)$ group, while the remaining eighteen fields X_μ which also transform as vector fields in four dimensions do not have massless modes. Similarly the thirty five fields A_5^a which transform as four dimensional scalars do not possess zero modes. Since we are working in the context of a supersymmetric model appropriate boundary conditions must also be imposed on the other fields in the higher dimensional gauge supermultiplet. What are these? In addition to the five dimensional vector field A_M, a five dimensional gauge supermultiplet consists of a symplectic Majorana spinor λ_i and a real scalar σ. The boundary conditions for these fields are given by the same equation (3.28), with $\lambda_{1+} = \frac{1}{2}(1+\gamma_5)\lambda_1$ transforming like A_μ while σ and $\lambda_{2+} = \frac{1}{2}(1+\gamma_5)\lambda_2$ transform like A_5. The zero modes of A_μ and λ_{1+} form a 4D $N=1$ gauge multiplet, while the other fields have no massless modes.

In such a framework supersymmetric chiral multiplets in the four dimensional effective theory can emerge in two ways - from five dimensional hypermultiplets propagating in the bulk of the space or from four dimensional chiral multiplets constrained to the boundaries of the five dimensional space. Consider first a five dimensional hypermultiplet Ψ transforming in the fundamental of SU(6) . Ψ consists of a Dirac fermion ψ and two complex scalars ϕ and ϕ^c. The boundary conditions on the scalars are,

$$\phi(x^\mu, x_5) = +\phi(x^\mu, -x_5) = \mathcal{C}\, Z\, \phi(x^\mu, x_5 + 2\pi R)\,, \tag{3.29}$$
$$\phi^c(x^\mu, x_5) = -\phi^c(x^\mu, -x_5) = \mathcal{C}\, Z\, \phi^c(x^\mu, x_5 + 2\pi R)\,, \tag{3.30}$$

where as before $Z = \mathrm{diag}(1,1,1,-1,-1,-1)$, and $\mathcal{C} = \pm 1$ is the parity of the field Ψ. Now under $\mathrm{SU}(3)\times\mathrm{SU}(3)$ the components of ϕ and ϕ^c transform as $[(3,1),(1,3)]$. As a consequence of these boundary conditions ϕ^c does not have any massless mode at all, while between the (3,1) component and the (1,3) component of ϕ only one has a massless mode while the other does not. The fields $\psi_+ = \frac{1}{2}(1+\gamma_5)\psi$ and $\psi_- = \frac{1}{2}(1-\gamma_5)\psi$ obey the same boundary conditions as ϕ and ϕ^c, respectively. Therefore ψ_-, like ϕ^c has no massless modes. The fields ϕ and ψ_+ each have a massless mode which together form a four dimensional $N=1$ chiral multiplet that transforms either as (3,1) **or** as (1,3) under the unbroken $\mathrm{SU}(3)\times\mathrm{SU}(3)$ gauge group. What about bulk hypermultiplets transforming under other representations of SU(6) ? These also lead in general to massless chiral multiplets which do not transform as complete representations of SU(6) but only as complete representations of the unbroken subgroup.

We now consider the case where chiral multiplets emerge from fields localized to the boundaries of the space at $y=0$ or $y=\pi R$. The point $y=\pi R$ is called the '3-3-1' point since the wave functions of all the X_μ gauge bosons vanish there. Fields localized to this point need not necessarily be in complete representations of SU(6) but need only be in complete representations of the unbroken group $\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)$. Similarly interactions localized to this point need only be invariant under the unbroken group. However fields localized to the point $y=0$ must be in complete representations of SU(6) , and interactions at this point must also be invariant under the full SU(6) gauge symmetry.

JHEP10(2004)036

We are now in a position to explain the matter content of the model. It consists of three generations of fields transforming under SU(6) as

$$\bar{6}_{-1}\ \bar{6}_{-3}\ 15_1\ 1_5\ 1_3\ 1_1 \tag{3.31}$$

where the subscripts give the charges of the various fields under the additional $\mathrm{U}(1)_{\mathrm{E}}$. In addition to this matter content there are two fields each transforming under SU(6) as the 20 dimensional representation. They have charges +1 and -1 under $\mathrm{U}(1)_{\mathrm{E}}$. It is straightforward to verify that matter content with this set of charge assignments is free of any anomalies.

We now turn to the Higgs content. It consists of two fields in the fundamental and anti-fundamental of SU(6) which break SU(6) to SU(5) , and which are uncharged under $\mathrm{U}(1)_{\mathrm{E}}$. The VEVs of these fields are denoted by $\langle 6\rangle_0$ and $\langle\bar{6}\rangle_0$. The Higgs fields responsible for the breaking of the global SU(3) symmetry and electroweak symmetry also emerge from the fundamental and antifundamental representations of SU(6) , and we denote these fields by 6_{H} and $\bar{6}_{\bar{\mathrm{H}}}$. Their charges under $\mathrm{U}(1)_{\mathrm{E}}$ are -2 and +2 respectively. It is clear from this that the Higgs content is also anomaly free.

We now consider the locations of the various matter fields in the higher dimensional space. Both fields transforming as the 20 of SU(6) are hypermultiplets propagating in the bulk of the space. The boundary conditions project out some of the states in these representations so that the massless fields in the low energy effective theory transform under $\mathrm{SU}(3)\times\mathrm{SU}(3)$ as shown below.

$$\begin{aligned} 20_{+1} &\rightarrow (1,1)_{+1,\bar{\tau}}\quad (3,\bar{3})_{+1} \\ 20_{-1} &\rightarrow (1,1)_{-1,\tau^c}\ (\bar{3},3)_{-1}\,. \end{aligned} \tag{3.32}$$

The three generations of matter fields are all localized on the brane at $y=0$. This gives us an understanding of their standard model quantum numbers and the quantization of electric charge.

What about the Higgs fields? Where are they located in the extra dimension? We assume that the fields that break SU(6) to SU(5) propagate in the bulk of the higher dimensional space while the fields that break electroweak symmetry are localized to the brane at $y=0$. Doublet-triplet splitting is realized by having a fundamental and anti-fundamental of SU(6) in the bulk that have the same $\mathrm{U}(1)_{\mathrm{E}}$ quantum numbers as the electroweak Higgs fields on the brane. The boundary conditions on the bulk fields are such as to leave the color triplets light. A mass term between these bulk fields and the brane fields on the boundary at $y=0$ will then have the effect of giving mass to all the color triplets while leaving one set of fields with the quantum numbers of $\hat{H}_u$ and $\hat{H}_d$ light. These light fields are linear combinations of the original bulk and brane Higgs fields.

We now turn our attention to the interactions of this theory. We start with the top quark Yukawa coupling. The charges of the various fields under $\mathrm{U}(1)_{\mathrm{E}}$ imply that of the two 20 dimensional representations only one can contribute to the top Yukawa interaction below, which is localized at $y=\pi R$.

$$6_{-2}\ 15_1\ 20_1 \rightarrow \begin{cases} (1,3)(\bar{3},1)(3,\bar{3})\,, \\ (1,3)(1,\bar{3})(1,1)_{\bar{\tau}}\,. \end{cases} \tag{3.33}$$

As before interactions of the form

$$\langle\bar{6}\rangle_0 \;\; 15_1 \; \bar{6}_{-1} \rightarrow \begin{cases} \langle 1, \bar{3}\rangle \, (3,3)(\bar{3},1), \\ \langle 1, \bar{3}\rangle \, (1,\bar{3})(1,\bar{3}) \end{cases} \tag{3.34}$$

decouple states in the 15 and the $\bar{6}$ which are vector-like under both SU(5) and $\mathrm{U(1)_E}$. This is true of matter in all three generations. In addition there are now the interactions

$$\langle\bar{6}\rangle_0 \, \bar{6}_{-1} 1_1 \qquad \langle\bar{6}\rangle_0 \, \bar{6}_{-3} 1_3 \tag{3.35}$$

whose effect is to decouple the SU(5) singlets which are vector-like under $\mathrm{U(1)_E}$.

We now consider the terms which give masses of order $\hat{m}$ to the light fields in the two 20 dimensional representations. These take the form

$$\tilde{\mu} \; 20_{+1} \; 20_{-1} \, , \qquad \tilde{\mu} \, \frac{\langle 6\rangle_0}{\Lambda} \, 15_1 \; 20_{-1} \, , \tag{3.36}$$

where $\tilde{\mu}$ is of order $\hat{m}$. The net result of the interactions (3.33) and (3.36) is that the physical t^c, Q_3 and τ^c are linear combinations of states in the 15 and the two 20's.

All the down-type Yukawa couplings as well as the up-type Yukawa interactions for the first two generations emerge from couplings of exactly the form discussed in the previous subsection. Constraints on the model arising from SU(5) mass relations can be avoided if the matter fields on the brane with quantum numbers $\bar{6}_3$ mix with additional fields in the bulk with the same quantum numbers, so that the light fields are linear combinations of bulk and brane fields. Alternatively, the first two generations of matter can emerge from bulk fields rather than brane localized fields without significantly affecting any of the other physics we have discussed.

It remains to consider the neutrino masses. How do they arise in this model? Perhaps the simplest possibility is that the neutrinos acquire Dirac masses with the 1_5 standard model singlet state. The smallness of the neutrino masses can be understood if a term of this form is forbidden until supersymmetry is broken, in analogy with the μ term [17].

$$\tilde{\mu} \, \frac{6_2}{\Lambda} \, 1_5 \; \bar{6}_{-3} \rightarrow \frac{\tilde{\mu}}{\Lambda} [1,2]_{\mathrm{H}} [1,1][1,2] \, . \tag{3.37}$$

We conclude from this discussion that it is indeed possible to construct realistic little supersymmetric models in which the coupling constants unify.

4. Conclusions

We have proposed 'little supersymmetry' as a solution to the supersymmetric little hierarchy problem. This involves extending the Yukawa coupling of the top quark to make it invariant under a global symmetry, of which the $\mathrm{SU(2)_L}$ subgroup is gauged. The physical Higgs field is the pseudo-Goldstone boson associated with the breaking of this global symmetry, and it can therefore naturally be significantly lighter than the other scalar superpartners. We have shown that is possible to embed the gauge and global symmetries together into a single grand unifying group, so that the prediction of gauge coupling constant unification can be maintained.

These models exhibit several distinctive features. In this scenario supersymmetry breaking at low energies is generally preferred. A firm prediction is the existence of new states at or close to the TeV scale that are required by the global symmetry. In the simplest unified models these have the quantum numbers of the 10 and $\overline{10}$ representations of SU(5) . Another characteristic feature is the existence of a Z' gauge boson with mass in the 300 GeV to a TeV range, which has direct couplings to quarks and leptons. There are also hints as to the superpartner spectrum. It is most natural for all the scalar superpartners to be significantly heavier than the lightest neutral Higgs. Further the electroweak gauginos are constrained to be lighter than the stops. In contrast to most other models which address the supersymmetric little hierarchy problem, the lightest neutral Higgs is expected to be lighter than 130 GeV just as in the MSSM.

How well does this proposal address the supersymmetric little hierarchy problem? This clearly depends on the details of the superpartner spectrum. Preliminary numerical results in a gauge mediated model seem to indicate that the fine-tuning is ameliorated by as much as an order of magnitude, from the 1-2 % level to about the 10% level. Detailed investigation into this and other models is currently underway [18].

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JHEP10(2004)036